On The Natural Origin of Baryons, Short-Range Mesons, and QCD Confinement, from Maxwell's Magnetic Equations for a Yang-Mills Field

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Abstract:

We demonstrate how the existence of baryons, that is, strongly-interacting sources consisting of exactly three fermion constituents, is a natural consequence of Maxwell's equation for a magnetic three-form $P = dF = d(dG + igG^2) = igdG^2$, with $dd = 0$, where $F = dG + igG^2$ is a Yang-Mills (non-Abelian) field strength two-form, G is a Yang-Mills vector boson (e.g., gluon) one-form, and g is the group charge strength. In particular, $P = igdG^2$ is shown to naturally consist of exactly three fermion constituents, irrespective of the chosen Yang-Mills group. The baryon charge B, over the finite spatial expanse of a baryon, is shown to be formed out of the volume integral of P, namely, $gB = \iiint P = i\iiint gdG^2$. Pauli exclusion among the three fermions within B is then enforced by choosing the specific Yang-Mills gauge group $SU(3)_{QCD}$. Quark and gluon confinement, and the mediation of nuclear interactions by short-range mesons, arises via the application of Gauss' law to a baryon, via $gB = \iiint P = \iint dG + i\iint gG^2$. If one considers the same analysis in the context of string theory, one may also by exclusion arrive at the weak SU(2) phenomenology, and other results of interest in accord with observed nuclear and atomic structure, and elementary fermion phenomenology including fermion generation replication.

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1. Introduction

In this paper, we pose the question "why, theoretically, do there exist in nature, naturally-occurring sources, namely 'baryons,' consisting of exactly three strongly-interacting fermion constituents?" The two most-common types of baryon, of course, are the proton and neutron.

We do know, because there are three quarks (partons) per baryon, to employ the Yang-Mills color group $SU(3)_{QCD}$ with a wavefunction $\psi^T = (R \quad G \quad B)$ in the fundamental representation, to ensure Fermi-Pauli-Dirac exclusion. But this does not explain why there are three quarks per baryon, and not some different number. If nature were to provide 4 or 7 or 11, for example, then we would merely force exclusion with SU(4) or SU(7) or SU(11) instead, and would still be asking "why?" with respect to that different number.

Maxwell's equations for magnetic charges may be written in terms of a magnetic three-form, as $P = dF = ddA = 0$, where $F = dA$ is the field strength two-form and A the potential one-form with $dd = 0$ for any two successive exterior derivatives. We demonstrate here that that when applied to a Yang-Mills field $F = dG + igG^2$, where G is a potential one-form and g is a group charge strength, the now-non-zero magnetic three-form $P = dF = d(dG + igG^2) = igdG^2$ turns out to consist of exactly three fermions, no mater what the rank of the chosen Yang-Mills group. $SU(3)_{QCD}$ is then motivated simply by exclusion within this three-fermion object, while the baryon number B is specified by the volume-integrated three-form $gB = \iiint P = i\iiint gdG^2$.

Then, applying Gauss' law, we show how gluon confinement is specified by $\iint dG = 0$, and quark confinement and the mediation of nuclear interactions by mesons which turn out to be short-range, is specified by $gB = i\iint gG^2$, where the surface integral is specified at the confinement barrier. If one considers the same analysis in the context of string theory, one may also by exclusion arrive at the weak SU(2) phenomenology, and other results all in accord with observed nuclear and atomic structure and elementary particle phenomenology, including fermion generation replication.

2. Magnetic Sources in Yang-Mills Gauge Theory

It is well known, and can be found in virtually any elementary textbook on particle physics or quantum field theory e.g., [1], equation (14.40), that the field strength tensor for a Yang-Mills (non-Abelian) gauge theory is:

$$F^{i\,\mu\nu} = \partial^\mu G^{i\,\nu} - \partial^\nu G^{i\,\mu} - gf^{ijk} G_j^{\ \mu} G_k^{\ \nu} \tag{2.1}$$

where the $G_i^{\ \mu}$ are the gauge bosons (classical potentials) of whatever Yang-Mills group one is using (for instance, weak SU(2) or SU(3) QCD), f^{ijk} are the group structure constants, g is the group charge strength, and the Latin internal symmetry index $i = 1,2,3 \ldots N^2 - 1$ for SU(N) is raised and lowered with the unit matrix δ_{ij}. Multiplying (2.1) through by the group generators T^i, and employing the group structure $f^{ijk} T_i = -i[T^j, T^k]$, one can readily rewrite (2.1) as:

$$F^{\mu\nu} = \partial^\mu G^\nu - \partial^\nu G^\mu + ig[G^\mu, G^\nu], \tag{2.2}$$

where $F^{\mu\nu} \equiv T^i F_i^{\mu\nu}$ and $G^\mu \equiv T^i G_i^\mu$ are NxN matrices for SU(N). Multiplying through by $dx_\mu dx_\nu$, and using the forms $G = G^\mu dx_\mu$, $F = \frac{1}{2!} F^{\mu\nu} dx_\mu \wedge dx_\nu = F^{\mu\nu} dx_\mu dx_\nu$, $G^2 = [G,G] = \frac{1}{2!}[G^\mu, G^\nu] dx_\mu \wedge dx_\nu = [G^\mu, G^\nu] dx_\mu dx_\nu$, $dG = \partial^\mu G^\nu dx_\mu \wedge dx_\nu = (\partial^\mu G^\nu - \partial^\nu G^\mu) dx_\mu dx_\nu$, in well-known fashion, this further compacts to (see [2], Chapter (4.5)):

$$F = dG + igG^2. \tag{2.3}$$

Starting with (2.2), let us now form the third-rank antisymmetric tensor $P^{\sigma\mu\nu} \equiv T^i P_i^{\sigma\mu\nu}$ for what is colloquially referred to as a "magnetic charge," as such:

$$\begin{aligned} P^{\sigma\mu\nu} &= \partial^\sigma F^{\mu\nu} + \partial^\mu F^{\nu\sigma} + \partial^\nu F^{\sigma\mu} = ig\left(\partial^\sigma [G^\mu, G^\nu] + \partial^\mu [G^\nu, G^\sigma] + \partial^\nu [G^\sigma, G^\mu]\right) \\ &= ig\left([\partial^\sigma G^\mu, G^\nu] + [G^\mu, \partial^\sigma G^\nu] + [\partial^\mu G^\nu, G^\sigma] + [G^\nu, \partial^\mu G^\sigma] + [\partial^\nu G^\sigma, G^\mu] + [G^\sigma, \partial^\nu G^\mu]\right). \end{aligned} \tag{2.4}$$

Using the magnetic three-form $P = \frac{1}{3!} P^{\sigma\mu\nu} dx_\sigma \wedge dx_\mu \wedge dx_\nu = P^{\sigma\mu\nu} dx_\sigma dx_\mu dx_\nu$, as well as $dF = \frac{1}{2!}\partial^\sigma F^{\mu\nu} dx_\sigma \wedge dx_\mu \wedge dx_\nu = (\partial^\sigma F^{\mu\nu} + \partial^\mu F^{\nu\sigma} + \partial^\nu F^{\sigma\mu}) dx_\sigma dx_\mu dx_\nu$ and also $dG^2 = \frac{1}{2!} d^\sigma [G^\mu, G^\nu] dx_\sigma \wedge dx_\mu \wedge dx_\nu = \left(\partial^\sigma [G^\mu, G^\nu] + \partial^\mu [G^\nu, G^\sigma] + \partial^\nu [G^\sigma, G^\mu]\right) dx_\sigma dx_\mu dx_\nu$, equation (2.4) can be multiplied through by $dx_\sigma dx_\mu dx_\nu$ and then expressed in compacted form:

$$P = dF = d\left(dG + igG^2\right) = igdG^2 = ig\left([dG, G] + [G, dG]\right). \tag{2.5}$$

Above, though we have employed $dd = 0$, a residual, non-zero self-interaction term $igdG^2$ remains.

Now, for a U(1) interaction such as electromagnetism, which omits the non-linear term G^2, and because $dd = 0$, we of course have $P = dF = 0$, which is Maxwell's magnetic equation, and which is often taken to state that there are no magnetic charges, only electric ones. Mathematically, we may state that F is a closed form in Abelian gauge theory, but that in Yang-Mills theory, F is open, and so, importantly, gives rise upon further differentiation to the non-zero $P^{\sigma\mu\nu}$ in (2.4), and the non-zero three-form P in (2.5).

Because we know of at least two interactions – weak and strong – where Yang-Mills gauge groups are in accord with observed physical reality, one should expect to come upon the non-zero magnetic three-forms $P = igdG^2$ of (2.5) for both interactions. T'hooft & Polyakov [3] and others have previously pointed out that Yang-Mills field theory seems to give rise to magnetic monopoles, but to date, no connection has been made from this line of inquiry to anything which has been experimentally observed. So, it behooves us to ask: "might these magnetic three-forms represent anything we have ever observed in the physical world?"

Because P is formed after taking two derivatives of the gauge potentials G (with $dd = 0$ dropping out but $igdG^2$ remaining), it is a "source" in the same sense as the current density four-vector specified by Maxwell's equation $J^\nu = \partial_\mu F^{\mu\nu} = \partial_\mu \partial^\mu A^\nu$; $\partial_\mu A^\mu = 0$ for "electric sources." As we now show, the integral objects $B = \iiint P = \iiint dF = i\iiint gdG^2$, may well represent the baryons which serve as the foundation of nuclear matter, where B is the baryon number charge.

3. Further Development of Yang-Mills Magnetic Sources

In this section, we shall perform some calculations which will enable us to connect the three-form P to baryons, especially on consideration of the resulting Feynman diagrams. Starting with (2.4), first, let us work with the terms $\partial^\sigma G^\mu$. Here, we employ the quantum mechanical operator equation:

$$\partial^\sigma G^\mu = i\left[q^\sigma, G^\mu\right], \tag{3.1}$$

see, for example, [4], just after equation (2.164). Substituting the above into (2.4) yields:

$$P^{\sigma\mu\nu} = -g\left(\left[\left[q^\sigma, G^\mu\right], G^\nu\right] + \left[G^\mu, \left[q^\sigma, G^\nu\right]\right] + \left[\left[q^\mu, G^\nu\right], G^\sigma\right] + \left[G^\nu, \left[q^\mu, G^\sigma\right]\right] + \left[\left[q^\nu, G^\sigma\right], G^\mu\right] + \left[G^\sigma, \left[q^\nu, G^\mu\right]\right]\right). \tag{3.2}$$

If we expand the commutators in the above, terms of the form $G^\mu q^\sigma G^\nu - G^\mu q^\sigma G^\nu$ appear throughout, so that all terms with q^σ sandwiched between the two G^μ drop out. Then, re-consolidating the commutators, (3.2) reduces to:

$$P^{\sigma\mu\nu} = g\left(+\left[\left[G^\mu, G^\nu\right], q^\sigma\right] + \left[\left[G^\nu, G^\sigma\right], q^\mu\right] + \left[\left[G^\sigma, G^\mu\right], q^\nu\right]\right). \tag{3.3}$$

Multiplying through by $dx_\sigma dx_\mu dx_\nu$, and using $G^2 = \left[G^\mu, G^\nu\right] dx_\mu dx_\nu$ and $P = P^{\sigma\mu\nu} dx_\sigma dx_\mu dx_\nu$, see above (2.3) and (2.5), as well as $q = q^\mu dx_\mu$, the above compacts to:

$$P = 3g\left[G^2, q\right] \qquad \left(= g\left(+\left[G^2, q^\sigma\right]dx_\sigma + \left[G^2, q^\mu\right]dx_\mu + \left[G^2, q^\nu\right]dx_\nu\right)\right). \tag{3.4}$$

The factor of 3 in the above is the first sign of a baryon. It is important to note that this factor of 3 does not at all depend on the specific choice of Yang-Mills group, that is, one does not need to posit SU(3) for the factor of 3 to emerge naturally in (3.4). In fact, it arises because Maxwell's magnetic equation, in tensor form, has three additive terms in order to form the antisymmetric third-rank tensor $P^{\sigma\mu\nu}$.

Now, let us work with the potentials G^μ in (3.3). Absent a longitudinal degree of freedom through some spontaneous symmetry breaking mechanism, we take the G^μ to represent massless vector bosons, analogous to the photon A^μ which mediates electromagnetic interactions, and like the gluons which we believe mediate strong interactions between quarks. In QED, one typically starts with Maxwell's equation for the electromagnetic current $J^\mu = \partial_\tau \partial^\tau A^\mu$ in covariant gauge $\partial_\mu A^\mu = 0$ with $A^\mu = \varepsilon^\mu e^{-iq^\tau x_\tau}$, and thereby establishes the relation $A^\mu = -\left(1/q^2\right)J^\mu$ between the A^μ and J^μ, where $q^2 = q^\sigma q_\sigma$ is the squared photon momentum. The J^μ in turn relates to a given fermion wavefunction ψ, for example, the electron or a quark, according to $J^\mu = \overline{\psi} Q \gamma^\mu \psi$ where Q is the U(1) electric charge generator and γ^μ are the Dirac matrices. It is also well-known that for massive rather than massless bosons, the term $1/q^2$ migrates to $1/\left(q^2 - M^2\right)$, where M is the vector boson mass. And, there are known methods for dealing with poles in $1/q^2$ or $1/\left(q^2 - M^2\right)$ (or other terms in propagators), for example, the $+i\varepsilon$ prescription which also bears a known relation to boson widths / lifetimes.

With this QED point of reference, let us analogously relate the G^μ, which we are taking to be massless, to their associated Yang-Mills current J^μ, using the relationship:

$$G^\mu = -\frac{1}{q_{(\mu)}{}^2} J^\mu = -\frac{1}{q_{(\mu)}{}^2} T^i \left(\overline{\psi}_{(\mu)} T_i \gamma^\mu \psi_{(\mu)}\right). \tag{3.5}$$

Here, $G^\mu \equiv T^i G_i{}^\mu$, $J^\mu = T^i J_i{}^\mu$, $J_i{}^\mu = \overline{\psi}_{(\mu)} T_i \gamma^\mu \psi_{(\mu)}$, and the ${}_{(\mu)}$ on the squared boson momentum $q_{(\mu)}{}^2 = q_{(\mu)}{}^\sigma q_{(\mu)\sigma}$ and the fermion wavefunction $\psi_{(\mu)}$ is a label, not an index, for later use, denoting the spacetime index of the boson G^μ and current J^μ with which it is associated. For a gauge group SU(N), the $\psi_{(\mu)}$ contains *N* Dirac spinors in the fundamental group representation.

We then return to (3.3), which, using the first two terms in (3.5), we may rewrite as:

$$P^{\sigma\mu\nu} = g\left(+ \frac{1}{q_{(\mu)}{}^2 q_{(\nu)}{}^2}\left[\left[J^\mu, J^\nu\right], q^\sigma\right] + \frac{1}{q_{(\nu)}{}^2 q_{(\sigma)}{}^2}\left[\left[J^\nu, J^\sigma\right], q^\mu\right] + \frac{1}{q_{(\sigma)}{}^2 q_{(\mu)}{}^2}\left[\left[J^\sigma, J^\mu\right], q^\nu\right]\right). \tag{3.6}$$

Using $J^2 = \frac{1}{2!}\left[J^\mu, J^\nu\right] dx_\mu \wedge dx_\nu = \left[J^\mu, J^\nu\right] dx_\mu dx_\nu = [J, J]$ and $q = q^\mu dx_\mu$, (3.6) compacts to:

$$P = g\left(\frac{q_{(\sigma)}{}^2 + q_{(\mu)}{}^2 + q_{(\nu)}{}^2}{q_{(\mu)}{}^2 q_{(\nu)}{}^2 q_{(\sigma)}{}^2}\right)\left[J^2, q\right]. \tag{3.7}$$

Contrast (3.4) where a factor 3 arose, to the above "Pythagorean" sum of three square momenta.

Then, inserting $J^\mu = T^i\left(\overline{\psi}_{(\mu)} T_i \gamma^\mu \psi_{(\mu)}\right) = \overline{\psi}_{(\mu)} T^i T_i \gamma^\mu \psi_{(\mu)}$ from (3.5) into (3.6), yields:

$$P^{\sigma\mu\nu} = g\begin{pmatrix} + \dfrac{1}{q_{(\mu)}{}^2 q_{(\nu)}{}^2}\left[\left[\overline{\psi}_{(\mu)} T^i T_i \gamma^\mu \psi_{(\mu)}, \overline{\psi}_{(\nu)} T^i T_i \gamma^\nu \psi_{(\nu)}\right], q^\sigma\right] \\ + \dfrac{1}{q_{(\nu)}{}^2 q_{(\sigma)}{}^2}\left[\left[\overline{\psi}_{(\nu)} T^i T_i \gamma^\nu \psi_{(\nu)}, \overline{\psi}_{(\sigma)} T^i T_i \gamma^\sigma \psi_{(\sigma)}\right], q^\mu\right] \\ + \dfrac{1}{q_{(\sigma)}{}^2 q_{(\mu)}{}^2}\left[\left[\overline{\psi}_{(\sigma)} T^i T_i \gamma^\sigma \psi_{(\sigma)}, \overline{\psi}_{(\mu)} T^i T_i \gamma^\mu \psi_{(\mu)}\right], q^\nu\right] \end{pmatrix}. \tag{3.8}$$

Finally, we add one more set of labels to (3.8). Taking, as an example, the top line of the above, let us work from right to left, and regard the right-most fermion $\psi_{(\nu)}$ to be in state "1," the middle fermions $\overline{\psi}_{(\nu)}$ and $\psi_{(\mu)}$ to be in state "2," and the left-most fermion $\overline{\psi}_{(\mu)}$ to be in state "3." We do the same for the other two lines as well. Thus, we now refer to the three distinct $\psi_{(\mu)}$, $\psi_{(\nu)}$ and $\psi_{(\sigma)}$, as the "μ","ν", and "σ" fermion wavefunctions, and to each such wavefunction being in state "1," "2," or "3." Thus, for example, $\psi_{(\mu 2)}$ designates the "μ" fermion in state "2." With this labeling, (3.8) now becomes:

$$P^{\sigma\mu\nu} = g\begin{pmatrix} +\frac{1}{q_{(\mu)}{}^2 q_{(\nu)}{}^2}\left[\left[\overline{\psi}_{(\mu 3)}T^i T_i \gamma^\mu \psi_{(\mu 2)}, \overline{\psi}_{(\nu 2)}T^i T_i \gamma^\nu \psi_{(\nu 1)}\right], q^\sigma\right] \\ +\frac{1}{q_{(\nu)}{}^2 q_{(\sigma)}{}^2}\left[\left[\overline{\psi}_{(\nu 3)}T^i T_i \gamma^\nu \psi_{(\nu 2)}, \overline{\psi}_{(\sigma 2)}T^i T_i \gamma^\sigma \psi_{(\sigma 1)}\right], q^\mu\right] \\ +\frac{1}{q_{(\sigma)}{}^2 q_{(\mu)}{}^2}\left[\left[\overline{\psi}_{(\sigma 3)}T^i T_i \gamma^\sigma \psi_{(\sigma 2)}, \overline{\psi}_{(\mu 2)}T^i T_i \gamma^\mu \psi_{(\mu 1)}\right], q^\nu\right] \end{pmatrix}. \tag{3.9}$$

Now, we are ready to make the connection to baryons.

4. The Theoretical Formulation of Baryons

We proceed to draw a Feynman diagram from (3.9), illustrated in Figure 1 below, using the following rules:

1) For the term $\left[\left[\overline{\psi}_{(\mu 3)}T^i T_i \gamma^\mu \psi_{(\mu 2)}, \overline{\psi}_{(\nu 2)}T^i T_i \gamma^\nu \psi_{(\nu 1)}\right], q^\sigma\right]$ in the top line of the above, we draw this as a fermion-fermion interaction between the current $\overline{\psi}_{(\nu 2)}T^i T_i \gamma^\nu \psi_{(\nu 1)}$ and the current $\overline{\psi}_{(\mu 3)}T^i T_i \gamma^\mu \psi_{(\mu 2)}$ mediated by the vector boson G^σ with momentum q^σ. We do the same for the other two terms of (3.9), thus producing three distinct Feynman diagrams for fermion-fermion interactions mediated by vector bosons.

2) Very importantly, we then interconnect all lines from all three terms. In particular, we make sure that each of $\psi_{(\mu 1)}$, $\psi_{(\mu 2)}$ and $\psi_{(\mu 3)}$, representing different states of the same fermion $\psi_{(\mu)}$, all reside on the same fermion line. Similarly for $\psi_{(\nu)}$ and $\psi_{(\sigma)}$, in each of states 1, 2, and 3. By following this rule, we find that q^μ naturally ends up on the opposite side of the diagram from $\psi_{(\mu)}$, and similarly for q^ν from $\psi_{(\nu)}$ and q^σ from $\psi_{(\sigma)}$.

3) We regard the presence of commutators throughout (3.9) as indicating that directional lines on the Feynman diagrams can be drawn in either direction relative to a given line. Thus, e.g., from (3.6), in the term $\left[\left[J^\mu, J^\nu\right], q^\sigma\right] = \left[J^\mu, J^\nu\right]q^\sigma - q^\sigma\left[J^\mu, J^\nu\right]$, we take $\left[J^\mu, J^\nu\right]q^\sigma$ to indicate a directional arrow showing q^σ propagation from J^ν to J^μ, we take $-q^\sigma\left[J^\mu, J^\nu\right]$ to show reverse propagation of q^σ from J^μ to J^ν, and we take the overall $\left[\left[J^\mu, J^\nu\right], q^\sigma\right]$ to thereby indicate non-specificity of direction of q^σ. Similarly, we take $J^\mu J^\nu$ in $\left[J^\mu, J^\nu\right] = J^\mu J^\nu - J^\nu J^\mu$, to represent one set of directional arrows for the associated fermions $\psi_{(\mu)}$ and $\psi_{(\nu)}$, we take $-J^\nu J^\mu$ to represent reversed arrows, and we take the overall $\left[J^\mu, J^\nu\right]$ to indicate non-specificity of direction of propagation for the fermions $\psi_{(\mu)}$, $\psi_{(\nu)}$.

4) We place $T^i T_i \gamma^\mu$ at the vertex between all of the $\psi_{(\mu)}$ and $\overline{\psi}_{(\mu)}$, we place $T^i T_i \gamma^\nu$ between all of the $\psi_{(\nu)}$ and $\overline{\psi}_{(\nu)}$, and we place $T^i T_i \gamma^\sigma$ between all of the $\psi_{(\sigma)}$ and $\overline{\psi}_{(\sigma)}$.

5) We draw a dashed line between state "3" and state "1", to represent an iterative "recycling" between state 3 and state 1. One may think of this a being in the nature of a "finite state machine" which iteratively cycles from states 1 → 2 → 3/1 → 2 → 3/1 → 2 → 3, or the reverse, depending on how one chooses directional arrows once those are added (note rule 3).

6) Using the diagram resulting from rules 1-5, we now remove the vector boson lines, and draw an equivalent second diagram showing only the fermions in the form of three interconnected, three-node, four-branch Mandelstam diagrams.

7) To establish the s, t, u scattering channels of this "three-node" Mandelstam diagram, we now draw both diagrams with directional lines, wherein the lines on the fermions point forward from state 1 to state 2 to state 3 and then back again to state 1. As between any two fermions with their interaction mediated by a vector boson, this means that the Fermion lines will be automatically reversed relatively to one another. Thus for example, in the commentator term $\left[\left(\overline{\psi}_{(\mu 3)} T^i T_i \gamma^\mu \psi_{(\mu 2)}\right), \left(\overline{\psi}_{(\nu 2)} T^i T_i \gamma^\nu \psi_{(\nu 1)}\right)\right] q^\sigma$ of (3.9), if we take the right hand term $\overline{\psi}_{(\nu 2)} T^i T_i \gamma^\nu \psi_{(\nu 1)}$ to represent "forward" propagation of the $\psi_{(\nu)}$ fermion, then the left-hand term $\overline{\psi}_{(\mu 3)} T^i T_i \gamma^\mu \psi_{(\mu 2)}$ will automatically represent "backward" propagation of the $\psi_{(\mu)}$ fermion. This will be a very important feature, when we shortly examine how mesons arise. We label each node of the associated three-node Mandelstam diagram as "s," thereby establishing this choice of directional lines as the s-channel for all three nodes. The other channels t, u can then be arrived at in the usual manner, as can other diagrams from "crossing" various lines.

The resulting diagrams, representing equation (3.9), are below:

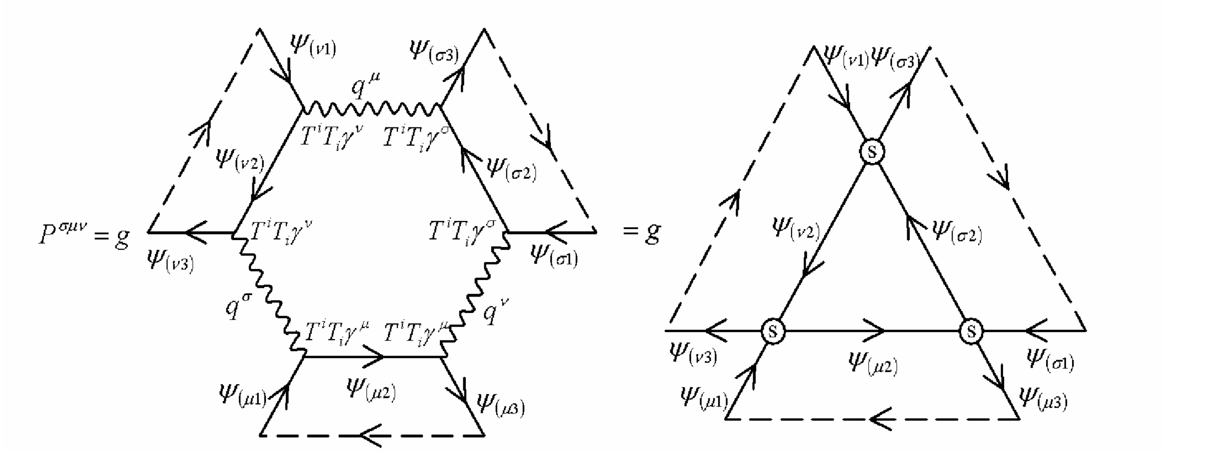

Figure 1

We now find that the Yang-Mills magnetic source $P^{\mu\nu\sigma}$ in (3.9) inherently contains exactly three fermion constituents, importantly, irrespective of the rank N>1 of the Yang-Mills gauge group. Nowhere in this derivation have we at any time had to assume that we were using SU(3). Might such a naturally-occurring, three-fermion source, be related to a baryon?

Having established a three fermion source, now let's proceed to make this "strong." Here, we turn to Fermi-Dirac statistics, and particularly, to the Pauli exclusion principle which will play a central role throughout the remaining development. Given the natural emergence of three fermions in Figure 1, and if we require that each of the three fermions in $P^{\mu\nu\sigma}$ possess a distinct quantum number for the purpose of satisfying Fermi-Dirac statistics / Pauli exclusion, then we are compelled to choose the group SU(3) to generate exclusion. So, now, we formally choose SU(3) as our Yang-Mills gauge group, and we assign all of the wavefunctions ψ in the above to the fundamental "color" representation of SU(3), $\psi^T = \begin{pmatrix} R & G & B \end{pmatrix}$, and choose as the group generators, the familiar 3x3 matrices $T^i = \frac{1}{2}\lambda^i$. We also assign $g \equiv g_s$, the strong charge strength. Note, however, that we do not assign color on a one-to-one correspondence with the spacetime index. Rather, we assign ${\psi_{(\mu)}}^T = \begin{pmatrix} R_{(\mu)} & G_{(\mu)} & B_{(\mu)} \end{pmatrix}$, ${\psi_{(\nu)}}^T = \begin{pmatrix} R_{(\nu)} & G_{(\nu)} & B_{(\nu)} \end{pmatrix}$, ${\psi_{(\sigma)}}^T = \begin{pmatrix} R_{(\sigma)} & G_{(\sigma)} & B_{(\sigma)} \end{pmatrix}$ so that each of the three fermions can assume the various color states with the overall set of interactions in Figure 1. The net result is a "colorless" baryon, as we shall see in the next section when we examine quark and gluon confinement.

In establishing $SU(3)_{QCD}$ in this way, we make an extremely useful pedagogical connection between spacetime and internal symmetry, using Pauli exclusion as a "bridge." Specifically, Maxwell's magnetic equation, coupled with Yang-Mills gauge theory, leads inexorably to objects consisting of exactly three fermions, as shown in Figure 1. Each of the fermions $\psi_{(\mu)}$, $\psi_{(\nu)}$ and $\psi_{(\sigma)}$, is associated with one of three spacetime indexes from their currents J^μ, J^ν, J^σ, respectively. Then, having three fermions, we choose SU(3) to enforce exclusion, and so come upon QCD in a very natural way, with the internal symmetry of strong interactions, $SU(3)_{QCD}$, emerging uniquely from the spacetime properties of $P^{\mu\nu\sigma}$, via exclusion. It bears emphasis again, that we arrived at Figure 1 solely by examining the spacetime properties of $P^{\mu\nu\sigma}$, without ever resorting to any assumption about any particular Yang-Mills gauge group, and then used exclusion as the bridge to choose SU(3). We shall return to this bridge in section 6 and thereafter, as it will point the way toward weak / strong unification, lepto-quark unification, and the structure of both atomic nuclei and atoms.

Now, we note that the magnetic tensor $P^{\mu\nu\sigma}$, like, for example, a current vector J^μ, is a density, locally defined. To describe a "whole" baryon, we will want to integrate the $P^{\mu\nu\sigma}$ density over a suitable spatial volume which we know from experiment is on the order of 1 Fermi^3. The fact that $P^{\mu\nu\sigma}$ is already, naturally, antisymmetric and of third rank, is perfect to establish a three form for the necessary volume integration, and directly lends credence to the fact that $P^{\mu\nu\sigma}$ represents a baryon, which is best understood with reference to integration over a finite, three-dimensional volume, just what one would expect. Thus, referring to (2.5), (3.4), and (3.7), we now integrate the baryon source density $P^{\mu\nu\sigma}$ over the 3-form $P = P^{\sigma\mu\nu} dx_\sigma dx_\mu dx_\nu$, to establish the formal, covariant expression for a whole baryon, including the quantized baryon number charge B = 1, 2, 3, . . . multiplied by the external charge strength g_s, as follows:

$$\begin{aligned} g_s B &= \iiint P = \iiint dF = \iiint d\left(dG + i g_s G^2\right) = i \iiint g_s dG^2 = 3 \iiint g_s \left[G^2, q\right] \\ &= \iiint g_s \left(\frac{{q_{(\sigma)}}^2 + {q_{(\mu)}}^2 + {q_{(\nu)}}^2}{{q_{(\mu)}}^2 {q_{(\nu)}}^2 {q_{(\sigma)}}^2} \right) \left[J^2, q\right] = \tfrac{1}{4} \iiint g_s \left(\frac{{q_{(\sigma)}}^2 + {q_{(\mu)}}^2 + {q_{(\nu)}}^2}{{q_{(\mu)}}^2 {q_{(\nu)}}^2 {q_{(\sigma)}}^2} \right) \left[\left[\overline{C}C, \overline{C}C\right], q\right] \end{aligned} \quad (4.2)$$

The final term, employing $J^2 = \frac{1}{4}\left[\overline{C}C, \overline{C}C\right]$, is based on forming the specific SU(3)$_{QCD}$ currents $J^\mu = T^i J_i^{\ \mu} = \overline{\psi} T^i T_i \gamma^\mu \psi$, according to:

$$J = \tfrac{1}{2}\begin{pmatrix} \frac{2}{3}\overline{R}R - \frac{1}{3}\overline{G}G - \frac{1}{3}\overline{B}B & \overline{G}R & \overline{B}R \\ \overline{R}G & -\frac{1}{3}\overline{R}R + \frac{2}{3}\overline{G}G - \frac{1}{3}\overline{B}B & \overline{B}G \\ \overline{R}B & \overline{G}B & -\frac{1}{3}\overline{R}R - \frac{1}{3}\overline{G}G + \frac{2}{3}\overline{B}B \end{pmatrix} \equiv \tfrac{1}{2}\overline{C}C, \tag{4.3}$$

where $\overline{C}C$ above compactly represents the illustrated $\overline{3} \times 3$ color matrix, and where, for a Dirac spinor in color state R, we define the one-form $\overline{R}R \equiv \overline{R}\gamma^\mu R dx_\mu$, and similarly for other colors. Because $\left[\overline{C}C, \overline{C}C\right]$ is a 3x3 matrix product of two 3x3 currents (4.3), the baryon (4.2) is, formally speaking, also a 3x3 matrix, transforming under the SU(3)$_{QCD}$ as $\left[\overline{3}3, \overline{3}3\right]$.

It is also noteworthy that the scalar (number) term $g_s\left(\dfrac{q_{(\sigma)}{}^2 + q_{(\mu)}{}^2 + q_{(\nu)}{}^2}{q_{(\mu)}{}^2 q_{(\nu)}{}^2 q_{(\sigma)}{}^2}\right)$ in (4.2), has its origin in the $1/q_{(\mu)}{}^2$ of (3.5), and that $1/q_{(\mu)}{}^2$ is ordinarily part of a propagator term. Thus, it may be worth further investigation to ascertain if this scalar term in (3.7) is somehow connected to the propagator for a baryon, and may in some way also be helpful in understanding baryon, e.g., proton and neutron, rest masses.

It must be stated that so far, we have only described a baryon in terms of its SU(3)$_{QCD}$ properties, but that this is not yet a real observed baryon, say, a proton or a neutron, because we have not yet addressed the question of from whence the SU(2)$_W$ weak isospin quantum number I^3 originates. The emergence of SU(2)$_W$ via exclusion, will be the subject of section 6.

Most importantly, for the moment, these three fermions are all naturally bound together in a single, integrated source, emanating from Maxwell's magnetic equations, and so the question "why are quarks confined?" has the simple, implicit answer that they are merely the individual constituents of the naturally-occurring, inseparable, three-fermion system specified in (4.2). Let us now explore further, what the foregoing may explicitly teach us about quark and gluon confinement, as well as what we can learn about the only entities which are not confined, namely, the short-range, short-lifetime mesons which bind together the atomic nucleus.

5. Quark and Gluon Confinement, Mesons, and Short-Range Nuclear Interactions

The goal of this section, based on the above development of a baryon, is to achieve confinement in a manner analogous to the so-called "MIT Bag Model" [5], [6] by paying close attention to what does and does not flow across the confinement surface of a baryon, but without an ad-hoc backpressure, and in a way that explains why the nuclear interaction is mediated by mesons. Many others have also made various efforts to solve the confinement problem, e.g., [7], [8], [9], [10], [11], [12], [13], [14], [15]. Nambu first realized that colored magnetic monopoles, if placed in a QCD vacuum with superconducting properties, would form flux tubes due to Meissner effect, which could help to explain confinement. [16]

To approach confinement, we take Maxwell's equations as our point of reference. Using differential forms, these are of course given by $*J = d*F = d*dA$ and $P = dF = ddA = 0$, with $F = dA$. The "*" denotes "duality," defined in tensor notation as $*F^{\mu\nu} = \frac{1}{2!}\varepsilon^{\mu\nu\sigma\tau}F_{\sigma\tau}$ for the field strength and $*J^{\sigma\mu\nu} = \varepsilon^{\sigma\mu\nu\tau}J_{\tau}$ for the current density, where $\varepsilon^{\mu\nu\sigma\tau}$ is the totally-antisymmetric Levi-Civita tensor. (The duality formalism was first developed by Reinich [17], and later elaborated by Wheeler, see [18], and [19], sections 3 and 4.) To write these equations in integral form, we use Gauss' law for a given p-form *H*, namely, $\int_{d} dH = \int_{d-1} H$, where d is the dimensionality of the closed surface over which the integration takes place. Thus, Maxwell's integral equations may be, and often are, written as:

$$eQ = \iiint *J = \iiint d*F = \iiint d*dA = \iint *F = \iint *dA, \tag{5.1}$$

$$0 = \iiint P = \iiint dF = \iiint ddA = \iint F = \iint dA. \tag{5.2}$$

In (5.1), Q is the total electric charge contained within the three-dimensional volume of integration, *e* is the running charge strength, and $\iint *F$ is the total (net) electric field flux through the closed two-dimensional surface of the integration volume. $\iint *F$ is equal to *Q*, which, of course, is quantized. In (5.2), because of $ddA = 0$, the total "magnetic charge" contained within the three-dimensional volume is zero, and the net magnetic field flux through the closed two-dimensional surface of that volume, is also equal to zero.

Baryons, insofar as they are understood at this time, present a "hybrid" of features from both equations (5.1) and (5.2). They are similar to (5.1), $eQ = \iiint *J$, insofar as the total baryon charge *B* contained within the baryon volume is non-zero and quantized. They are similar to (5.2), $\iint F = \iint dA = 0$, however, because despite a non-zero charge within the baryon volume, there are no gluons flowing across the boundary of the baryon volume. It is as if there is an electric charge inside the surface of integration, yet one can nevertheless shut down the electric field and the flow of photons through that surface. Further there are no currents of individual quarks flowing across the boundary. What does flow across the boundary, however, are color-neutral mesons with extremely short range and short lifetime, consisting of quark / anti-quark pairs, which bind nucleons together to form atomic nuclei.

Now, to show confinement, and a nexus to strong, nuclear interactions, one would need to establish five points, i.e., "legs of confinement": 1) a non-zero baryon charge occupying the volume within the confinement boundary; 2) no gluons flowing across the boundary; 3) mesons, i.e., quark / antiquark pairs, which do flow across the boundary; 4) a very short range and lifetime for these mesons, i.e., mesons which are very unstable; and 5) no individual quarks flowing across the boundary. We shall address these in sequence.

To start, let us apply Gauss' law to see what (4.2) teaches us about what does and does not flow through the confinement boundary of a baryon. In particular, we write a set of integral equations similar to (5.1) and (5.2), and we regard the integration surface to be the outer confinement surface of the baryon, that is, the ~1 Fermi barrier within which gluons and quarks, i.e., color charges, are thought to be confined. By Gauss, and with $ddG = 0$, we write (4.2) as:

$$g_s B = \iiint P = \iiint dF = \iiint d\left(dG + ig_s G^2\right) = i\iiint g_s dG^2 = \iint F = \iint dG + i\iint g_s G^2 \, . \tag{5.3}$$

Now, in (5.3), B is a scalar quantity which represents the baryon number of the baryon, similar in nature to electric charge Q. In general, B=1,2,3… is quantized, similarly to Q. Equation (5.3) says that we do have a non-zero baryon charge inside the confinement surface. That is "leg 1." For the moment, we consider a single baryon, B=1.

Now, we examine the sub-equation $i\iiint g_s dG^2 = \iint dG + i\iint g_s G^2$ in the above, where we purposely separate the surface integral into two terms. By Gauss, $\iiint dG^2 = \iint G^2$, we deduce:

$$\iint dG = 0 \, . \tag{5.4}$$

This is very similar to Maxwell's magnetic equation $\iint dA = 0$ from (5.2), this may be interpreted to state that there is no colored field flux, i.e., no gluon flux, across the confinement surface. This is the second leg of confinement (point 2).

With (5.4) above, (5.3) reduces to:

$$g_s B = i\iint g_s G^2 \, , \tag{5.5}$$

which yields the remaining three legs of confinement, and which we shall now explore in detail.

First, expanding with $G^2 = \left[G^\nu, G^\mu\right] dx_\nu dx_\mu$, we again use (3.5) to write:

$$\left[G^\nu, G^\mu\right] = \frac{1}{q_{(\nu)}^{\;2}\, q_{(\mu)}^{\;2}}\left[J^\nu, J^\mu\right] = \frac{1}{q_{(\nu)}^{\;2}\, q_{(\mu)}^{\;2}}\left[\overline{\psi}_{(\nu 3)} T^i T_i \gamma^\nu \psi_{(\nu 2)}, \overline{\psi}_{(\mu 2)} T^i T_i \gamma^\mu \psi_{(\mu 1)}\right], \tag{5.6}$$

where we have switched the μ,ν indexes to $\left[G^\nu, G^\mu\right] = -\left[G^\mu, G^\nu\right]$ to facilitate momentary comparison to Figure 1, and where we have also introduced the same "1," "2," "3" states as previously. Contrasting, we see that (5.6) is very similar to each of the three main terms in (3.6) and (3.9). The differences to note, however, are that (5.6) only contains one such term, not three, and that the term is only $\left[J^\nu, J^\mu\right]$, not $\left[\left[J^\nu, J^\mu\right], q^\sigma\right]$, i.e., there is no commutation relationship with a gluon momentum q^σ.

Now, let see if we can draw a Feynman diagram for the last term in (5.6), similarly to how we drew Figure 1 earlier. It is helpful to compare (5.6) to the first term in each of (3.6) and (3.9), for the interaction between the $\psi_{(\mu)}$ and $\psi_{(\nu)}$ fermions. It is also helpful to compare the above-noted equations to the lower-left Mandelstam node in Figure 1, because that node illustrates this interaction between these $\psi_{(\mu)}$ and $\psi_{(\nu)}$ fermions.

First, in Figure 2 below, we isolate from Figure 1, only that portion of Figure 1 which represents this $\psi_{(\mu)}$, $\psi_{(\nu)}$ interaction. Second, because there is no commutation relationship with a gluon momentum q^σ in (5.6), we place an "X" over the gluon line, and also, inside the Mandelstam node in place of the "s." Then, we simply take the Mandelstam part of the diagram, and draw the $\psi_{(\mu)}$ and $\psi_{(\nu)}$ lines parallel to one another. Finally, noting the relative orientation of the arrows as extracted from Figure 1, we reverse the lines for $\psi_{(\nu)}$, and then represent this as the anti-fermion $\overline{\psi}_{(\nu)}$. We end up, at the lower right of Figure 2, with an quark / anti-quark pair which looks just like how one might draw a meson. Mesons are known to mediate nuclear interactions, they are known to be very unstable with short range and lifetime, and they are known from numerous experiments to be the only type of particle which can penetrate a confinement surface. Multiplying (5.6) through by $q_{(\nu)}{}^2 q_{(\mu)}{}^2$, this is all illustrated below.

$$q_{(\nu)}{}^2 q_{(\mu)}{}^2 \left[G^\nu, G^\mu\right] = \left[J^\nu, J^\mu\right] = \left[\overline{\psi}_{(\nu 3)} T^i T_i \gamma^\nu \psi_{(\nu 2)}, \overline{\psi}_{(\mu 2)} T^i T_i \gamma^\mu \psi_{(\mu 1)}\right]$$

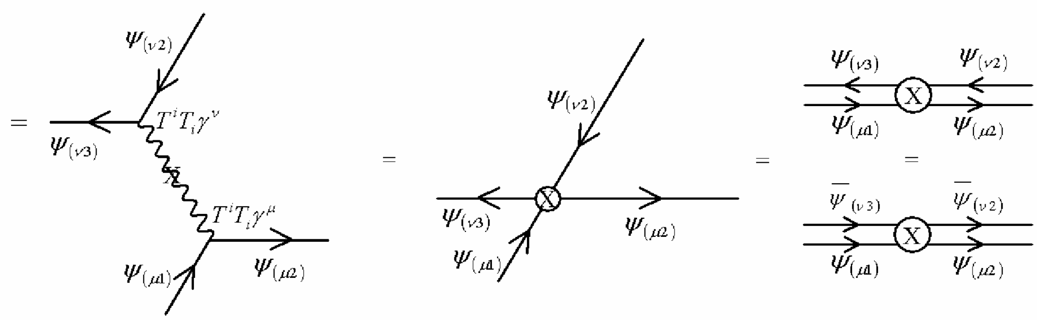

Figure 2

Reviewing Figure 2, is appears that the non-linear term $G^2 = \left[G^\nu, G^\mu\right] dx_\nu dx_\mu$, which is at the center of Yang-Mills gauge theory, in some manner represents a meson. We see too, that in $\left[J^\nu, J^\mu\right]$, the right-side current J^μ represents the quark current of the meson, and the left-side current J^ν represents the antiquark current of the meson. Let us now formalize this connection.

Going back to (5.5), the expression $g_s B = i \iint g_s \left[G^\nu, G^\mu\right] dx_\nu dx_\mu$ tells us that $ig_s\left[G^\nu, G^\mu\right]$ is what flows through the confinement barrier. If $ig_s\left[G^\nu, G^\mu\right]$ is representative of a meson, then $g_s B = i \iint g_s \left[G^\nu, G^\mu\right] dx_\nu dx_\mu$ is a simple declarative statement that "mesons flow through the confinement surface," which is the fourth leg of confinement. Pursuing this possibility, let us now define a second rank, antisymmetric, "meso-electromagnetic" field tensor:

$$M^{\nu\mu} \equiv \frac{ig_s}{q_{(\nu)}{}^2 q_{(\mu)}{}^2}\left[J^\nu, J^\mu\right] = ig_s\left[G^\nu, G^\mu\right]. \tag{5.7}$$

We may then use the above to rewrite (5.5), with $M = M^{\nu\mu} dx_\nu dx_\mu = ig_s G^2$, as:

$$g_sB = i\iint g_sG^2 = i\iint \frac{g_s}{q_{(\nu)}^2\, q_{(\mu)}^2} J^2 = i\iint g_s\left[G^\nu, G^\mu\right]dx_\nu dx_\mu = i\iint \frac{g_s}{q_{(\nu)}^2\, q_{(\mu)}^2}\left[J^\nu, J^\mu\right]dx_\nu dx_\mu \\ \equiv \iint M^{\nu\mu}dx_\nu dx_\mu = \iint M \quad . \tag{5.8}$$

Now, we focus on $g_sB = \iint M$, which is identical in form to Maxwell's equation (5.1), $eQ = \iint *F$. Very importantly, however, in the above, $M = ig_sG^2$, while in contrast, for Maxwell's equation (5.1), $F = dA$. If we define $M_i^{\nu\mu}$ according to $T^iM_i^{\nu\mu} \equiv M^{\nu\mu}$ for the $T^i = \frac{1}{2}\lambda^i$ of SU(3)$_{QCD}$, we may then couch this contrast with Maxwell's (5.1) even more pointedly by defining the components of $M_i^{\nu\mu}$ as:

$$M_i^{\nu\mu} \equiv \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{pmatrix}_i . \tag{5.9}$$

With $g_sB = \iint M$ and (5.9), we may then think of the strong interaction between baryons, classically, being mediated by a meso-electromagnetic field, where **E** is the "meso-electric" and the **B** is the "meso-magnetic" field. We may then follow Faraday by drawing a baryon with field lines emanating therefrom, precisely like for an electric charge, right down to the meso-magnetic field **B** arising from relative motion between baryons. With the first-rank dual $*P^\mu = \frac{1}{3!}\varepsilon^{\mu\nu\sigma\tau}P_{\nu\sigma\tau}$ of a baryon (see [19], equation (3.51)), one may even think about a classical Lorentz-force analog of the form $dp^\nu/d\tau = M^\nu{}_\mu *P^\mu$, where $p^\nu \equiv T^ip_i^\nu$ is a Yang-Mills, 3x3 momentum vector for a baryon density $P_{\nu\sigma\tau}$ under the influence of a meso-electromagnetic field $M^\nu{}_\mu$. However – and this becomes a critical point – for the electromagnetic interaction, $F = dA$, and this is an inverse-square, long-range interaction. For the nuclear interaction, in contrast, $M = ig_sG^2$, and this is a short range interaction. This is the only difference between $g_sB = \iint M$ and $eQ = \iint *F$. If the nuclear interaction is to be short range, then this short range must arise from the fact that $M = ig_sG^2$ for nuclear interactions, in contrast to the fact that $F = dA$ for electromagnetic interactions. How might this occur?

Now, focus on the other interesting difference between the $\left[J^\nu, J^\mu\right]$-only term in (5.6), (5.8) and Figure 2, and the $\left[\left[J^\nu, J^\mu\right], q^\sigma\right]$ terms in (3.6), (3.9) and Figure 1. The absence of any commutation with q^σ means that in contrast to Figure 1, there is no gluon mediating between the quark-antiquark pair in Figure 2. That is represented by the "Xs" in Figure 2. The absence of a commuting q^σ suggests that the meson will be a highly-unstable particle, because there is no vector boson mediator to permanently bind together the quark and antiquark. This is but a short-lived association, because if a quark and antiquark wish to "jet" out of the baryon together, they may do so, but they leave behind any gluons which might bind them together. Since a meson field $M^{\nu\mu} \sim \left[J^{\nu}, J^{\mu}\right]$ only is emitted and absorbed by a baryon in the form of $\left[J^{\nu}, J^{\mu}\right]$ currents, i.e., quark / anti-quark pairs, since there are no vector boson mediators binding the meson together (we leave open the prospect of scalar boson mediators which may provide a mass mechanism), and since the very existence of the $M^{\nu\mu}$ depends on the unbound $\left[J^{\nu}, J^{\mu}\right]$ currents remaining together, the meson will decay after a very short time. For example, if the meson is a $\sqrt{2}\pi^0 = \overline{\mu}u + \overline{d}d$ meson, it may decay into the vacuum via an electron-positron pair and gamma ray with a half-life of about 10^{-16} seconds. Or, it may after its brief run be absorbed back into a baryon, be it the original baryon, or a nearby baryon. In any event, while $g_s B = \iint M$ bears a clear resemblance to Maxwell's charge equation $eQ = \iint {}^*F$ insofar as how one would envision the meso-electromagnetic Faraday-type field lines surrounding a baryon, the unstable, unbound $M = ig_s G^2$, in contrast to the stable $F = dA$, causes the strong interaction to have extreme short range in contrast to the unlimited range of electromagnetism. $F = dA$ originates in a single vector particle. $M = ig_s G^2$ depends on an association of two fermions which have no vector bosons binding them together. The electromagnetic interaction is mediated by vector boons (photons) and is long-range and stable. The nuclear interaction is mediated by mesons which are highly unstable, and therefore, will live over only a very short range and lifetime. These are the third and fourth legs of confinement.

Finally we then ask, what about individual quarks? Can they flow across the confinement barrier also? Equation (5.8), specifically, $g_s B = i\iint \frac{g_s}{q_{(\nu)}{}^2 q_{(\mu)}{}^2} J^2$, says "no, they cannot." While currents of quarks J^{μ} can and do flow through the confinement barrier, they never do so alone, but only in pairs with an antiquark current J^{ν}, in the $J^2 = \left[J^{\nu}, J^{\mu}\right]$ configuration which underlies the mesons. No individual gluons G^{μ}. No individual currents J^{μ}. Only mesons $\sim \left[J^{\nu}, J^{\mu}\right]$ can be emitted or absorbed through the barrier of a baryon. This is the fifth and final leg of confinement.

It is also to be observed, that the confinement of individual quarks, in favor of mesons being allowed to penetrate the baryon surface, is enforced by the very structure of the differential forms spacetime geometry. If we are talking about flow across the confinement surface, we are talking about flow through area elements $dx_{\nu} dx_{\mu}$. This needs to be contracted with a second rank antisymmetric tensor, such as the field strength tensor $F^{\nu\mu}$ in Maxwell's equations, see (5.2), or the meson field $M^{\nu\mu}$ of (5.7), (5.9). Single-fermion currents are first or third rank-only, not second rank, see, e.g., (5.1) where we employ the current three-form ${}^*J^{\sigma\mu\nu} = \varepsilon^{\sigma\mu\nu\tau} J_{\tau}$. The best we can do, is contract the first-rank currents J^{ν}, J^{μ} with $dx_{\nu} dx_{\mu}$ in $\left[J^{\nu}, J^{\mu}\right]$ quark / antiquark meson pairs. In this way, the existence of mesons as the mediators of strong interactions, and the confinement of quark currents such that they cannot cross the barrier unless accompanied by an anti-quark currents, spring directly from the structure of the differential geometry. Confinement of quarks, and passage of mesons, is geometrically-mandated.

We may now proceed to encapsulate all of the above into very compact form, bearing a close resemblance with Maxwell's integral equations (5.1), (5.2), but with the crucial differences developed above. Because the baryon three-form is given by $P = ig_s dG^2$, equation (2.5), and the meson two-form is given by $M = ig_s G^2$, see (5.8), we may now relate baryons directly to mesons in differential form, by way of the very direct, simple equation (contrast $*J = d*F = d*dA$):

$$P = dM = ig_s dG^2 \ . \qquad (5.10)$$

In integral form, using Gauss' law, this becomes:

$$\boxed{g_s B = \iiint P = \iint M = i\iint g_s G^2 = i\iint \frac{g_s}{q_{(\nu)}^{\ 2} q_{(\mu)}^{\ 2}} J^2} \quad \text{versus} \quad eQ = \iiint *J = \iint *F = \iint *dA . \qquad (5.11)$$

Reproducing (5.4), and applying Gauss' law along with (2.3) as $dG = F - igG^2$ and $B = i\iint gG^2$ from above, one also has:

$$\boxed{\iiint ddG = \iint dG = \iint \left(F - ig_s G^2\right) = \iint F - g_s B = 0} \quad \text{versus} \quad \iiint ddA = \iint dA = \iint F = 0 . \qquad (5.12)$$

Although $M = ig_s G^2$ and $F = dG + ig_s G^2$ so that $F = M + dG$, the combination of $g_s B = \iint M$ from (5.11) and $g_s B = \iint F$ from (5.12) tells us, across the baryon surface, that:

$$\iint F = \iint M \ . \qquad (5.13)$$

From outside the baryon, the meso-electromagnetic field strength M is indistinguishable from the field strength F because $\iint dG = 0$. We can describe this in terms of a new, local gauge vector symmetry, as follows: If we write $F = M + dG$ as $F^{\mu\nu} = M^{\mu\nu} + \partial^{[\nu} G^{\mu]}$, then (5.11) and (5.12) are invariant under the local gauge transformation $F^{\mu\nu} \to F^{\mu\nu}{}' = F^{\mu\nu} - \partial^{[\nu} G^{\mu]} = M^{\mu\nu}$. Contrast this to both $A^\mu \to A^\mu{}' = A^\mu + \partial^\mu \Lambda$ and the gravitational $g^{\mu\nu} \to g^{\mu\nu}{}' = g^{\mu\nu} + \partial^{(\mu}\Lambda^{\nu)}$. In form language: the nuclear interaction (outside the baryons) is invariant under $F \to F' = F - dG = M$. Because of this, we can then transform to a gauge in which:

$$F = M \ , \qquad (5.14)$$

and thus identify the field strength F directly with the meso-electromagnetic M of (5.9). This invariance of the nuclear (baryon) interaction under $F \to F' = F - dG = M$, is another way of stating that nuclear interactions are colorless.

All of the essential features of confinement and nuclear interactions are encapsulated in (5.11) and (5.12) above. $g_s B = \iiint P$ is leg 1 of confinement – non-zero charge within the baryon. $\iint dG = 0$ is leg 2 – no gluon flux through the baryon surface. $g_s B = \iint M$ contains leg 3 – meson flux does occur. $g_s B = i\iint \frac{g_s}{q_{(\nu)}^{\ 2} q_{(\mu)}^{\ 2}} J^2$ contains legs 4 and 5 – the mesons are short range because there are no mediating vector bosons binding their individual currents, and individual quark flux does not occur because quarks only flow in colorless quark / antiquark current pairs J^2. Like Maxwell's equations for electromagnetism, (5.11) and (5.12), at bottom, are the underlying equations of nuclear physics.

While the development here has discussed five distinct "legs" of confinement, a different, but parallel formulation of this problem was posited by Jaffe and Witten for the so-called "mass-gap" problem. They state that "for QCD to describe the strong force successfully, it must have at the quantum level the following three properties, each of which is dramatically different from the behavior of the classical theory: (1) It must have a 'mass gap;' . . . (2) It must have 'quark confinement,' . . . (3) It must have 'chiral symmetry breaking,' . . ." They continue: "The first point is necessary to explain why the nuclear force is strong but shortranged; the second is needed to explain why we never see individual quarks; and the third is needed to account for the "current algebra" theory of soft pions that was developed in the 1960s." [20]

"Strong but shortranged" is leg 4. "Why we never see individual quarks" or gluons are legs 2 and 5. The "theory of soft pions," more generally, the existence of mesons as the mediators of strong interactions, is leg 3. Not specifically mentioned, but certainly implied, is leg 1: the baryon charge enclosed with the baryon surface is non-zero.

Before concluding, we make a very preliminary incursion into propagators and masses, leaving further detailed exploration for a separate paper. Contrasting (5.7) with (3.5), it now appears as if the meson propagators may be related to $\frac{ig_s}{q_{(\nu)}{}^2 q_{(\mu)}{}^2}$, which may in turn may relate to individual quark masses since there is a square momentum label for each quark current. Then, the term $g_s\left(\frac{q_{(\sigma)}{}^2+q_{(\mu)}{}^2+q_{(\nu)}{}^2}{q_{(\mu)}{}^2 q_{(\nu)}{}^2 q_{(\sigma)}{}^2}\right)=g_s\left(+\frac{1}{q_{(\mu)}{}^2 q_{(\nu)}{}^2}+\frac{1}{q_{(\nu)}{}^2 q_{(\sigma)}{}^2}+\frac{1}{q_{(\sigma)}{}^2 q_{(\mu)}{}^2}\right)$, that is, the sum of three prospective meson propagators, appears to relate to the baryon propagators, because it multiples the underlying baryon structure $\left[J^2, q\right]$ of Figure 1, contrast with (3.7). On careful, further consideration, these may allow a better understanding of the mass relationships among baryons, mesons, and quarks.

6. Protons, Neutrons, Deuterons and Dibaryons, and Weak / Strong Unification

In briefly talking about specific quarks such as *u* and *d* and specific mesons in the last section, we were really a bit ahead of ourselves, because we have thus far only justified $SU(3)_{QCD}$. To talk about specific, observed quarks and mesons, and even about, e.g., protons and neutrons, we must also motivate $SU(2)_W$ weak isospin. That is the subject of this section.

We have shown how Figures 1 and 2 represent baryons and mesons. We have shown how, after assigning $\psi^T = \begin{pmatrix} R & G & B \end{pmatrix}$ and $T^i = \frac{1}{2}\lambda^i$ to satisfy exclusion, we can use Figures 1 and 2 and their related equations to describe the strong interactions of quarks within baryons and even confinement and the short range QCD properties of mesons. But, we cannot, yet, talk about observed baryons or mesons. We can only talk about their QCD properties. Thus, to the next question: "How do we go from here, to describing the observed baryons and mesons, say, at least the proton and the neutron to start?"

This brings us to the doorstep of weak / strong unification, because that which distinguishes, e.g., a proton from a neutron or a $\pi^\pm$ from a π^0 has nothing to do with color, and everything to do with $SU(2)_W$ weak isospin, particularly, the third generator $I^3 = \pm\frac{1}{2}$ of $SU(2)_W$.

Also of interest is the electric charge $Q = Y + I^3$, however, this is not necessary for defining exclusive states within a baryon, or for distinguishing mesons, because all quarks have the same $Y = \frac{1}{6}$ and so I^3 will distinguish one baryon or meson from the next as surely as will Q. In the proton, p=uud, two of the quarks have isospin up, and the third has isospin down. In the neutron, n=udd, two of the quarks have isospin down, and the third has isospin up. The widespread, naturally-occurring deuteron = pn, which forms the nucleus of deuterium and is the most-common dibaryon, and from which the nuclei of more complex, non-isotopic atoms may be built, contains a total of six quarks. If we use $\Uparrow, \Downarrow$ to represent isospin up and down, respectively, the deuteron = pn may be thought to contain the six mutually-exclusive fermion states, $pn = (R\Uparrow, G\Uparrow, B\Downarrow), (R\Downarrow, G\Downarrow, B\Uparrow)$, for example. Finally, and the main point of all of this, is that $SU(3)_{QCD}$ alone cannot by itself get us from the baryon of Figure 1 to real protons and neutrons. We need, at the very least, the product group $SU(3)_{QCD}$ x $SU(2)_W$ (which the above discussion of the deuteron roughly represents), and even more preferably, we need to understand what unifies the weak and strong interactions.

This returns us to the Pauli exclusion principle. In Section 4, the spacetime configuration of a baryon yielded the three distinct component fermions $\psi_{(\mu)}$, $\psi_{(\nu)}$ and $\psi_{(\sigma)}$ in Figure 1. To enforce fermion exclusion, we were led to assign the internal symmetry of $SU(3)_{QCD}$, via $\psi^T = (R \quad G \quad B)$ and $T^i = \frac{1}{2}\lambda^i$, to each of these three fermions. This use of exclusion as a bridge between spacetime and internal symmetry has important pedagogical value for approaching other internal symmetries as well. For example: If we have been able to arrive naturally at three-fermion objects via Maxwell's equation $P = igdG^2$ for Yang-Mills magnetic sources and then at $SU(3)_{QCD}$ solely through the bridge of exclusion, is there some way to arrive at $SU(2)_W$ of weak isospin, also, solely through exclusion? Stated most simply: "can we uncover some situation which compels us to introduce $SU(2)_W$ to satisfy exclusion, as we did with $SU(3)_{QCD}$?" And, since $SU(3)_{QCD}$ arose via exclusion, from the spacetime properties of baryons, we also ask "is there a spacetime property of baryons which similarly compels $SU(2)_W$?" The motivation here, is that $SU(2)_W$ not be introduced *ad hoc*, but that it be motivated and, indeed, required, to enforce some necessary exclusion principle, rooted in spacetime itself.

The spacetime origin of $SU(3)_{QCD}$ above, in the end, can be summarized entirely from "counting" arguments, based on the third rank antisymmetric $P^{\sigma\mu\nu}$ containing exactly three first rank vector bosons G^μ, G^ν, G^σ, see (3.3), and three first rank currents J^μ, J^ν, J^σ, see (3.6), with three spacetime indexes, from which we obtained exactly three fermions $\psi_{(\mu)}$, $\psi_{(\nu)}$, $\psi_{(\sigma)}$ in need of exclusion. Thus, we ask: "is there a similar counting argument which can be gleaned for $SU(2)_W$?"

Surprisingly, $SU(2)_W$ appears to arise most naturally if we consider the very basic features of a string theory. Consider, for example, a closed string tracing out a world sheet $X^\mu(\tau,\sigma)$ in spacetime. The current associated with this string, , see, e.g., [2], pp. 222,223, is:

$$J^{\mu\nu}(x^\alpha) = \int d\tau d\sigma \det\begin{pmatrix} \partial_\tau X^\mu & \partial_\tau X^\nu \\ \partial_\sigma X^\mu & \partial_\sigma X^\nu \end{pmatrix} \delta^{(4)}\left[x^\alpha - X^\alpha(\tau,\sigma)\right] = \overline{\psi}_{(\mu\nu)}\sigma^{\mu\nu}\psi_{(\mu\nu)}, \tag{6.1}$$

where $\delta^{(4)}$ is the Dirac delta in four-dimensional spacetime, and where $\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right]$ is used in recognition that this second-rank, current $J^{\mu\nu}$ is antisymmetric and so would need to be composed out of antisymmetric combinations of the Dirac γ^{μ}. Similarly to how we proceeded earlier, we label the fermions with the (in this case, pair of) spacetime indexes $_{(\mu\nu)}$ of the associated current. Most importantly – and in fact the only point that matters for the exposition to follow – in such a string theory employing the 2x2 determinant in (6.1), "one" is added to the rank of every spacetime object in the associated antisymmetric field theory. The gauge potential $G^{\mu} \Rightarrow G^{\mu\nu}$, the field strength tensor $F^{\mu\nu} \Rightarrow F^{\mu\nu\sigma}$, and the magnetic charge (baryon) $P^{\sigma\mu\nu} \Rightarrow P^{\sigma\mu\nu\tau}$, all totally antisymmetric. Now, using these "plus one rank" objects, let's count some more.

The "string baryon" $P^{\sigma\mu\nu\tau}$ would be rank four. The "string currents" $J^{\mu\nu}$ would be rank two. Out of the four spacetime indexes in the string baryon, one can the form 6 = C (4,2) combinations, $\sigma\mu, \sigma\nu, \sigma\tau, \mu\nu, \mu\tau, \nu\tau$. That is, one can "populate" a "string baryon" with exactly six "string currents" for six fermions. Thus, a "string baryon" so-defined will contain a total of six fermions, $\psi_{(\sigma\mu)}, \psi_{(\sigma\nu)}, \psi_{(\sigma\tau)}, \psi_{(\mu\nu)}, \psi_{(\mu\tau)}, \psi_{(\nu\tau)}$, labeled in these same index-pair combinations. For exclusion of the fermions in such a six-component baryon, one would choose SU(6), and so there would be six different "colors" of fermion. However, the "string" nature of these fermions would only be apparent over very small distances, possibly within a few orders of magnitude of the Planck scale. For everyday observation, this SU(6) symmetry will break down into the SU(3) of QCD based on 3 = C (3,1) spacetime index combinations developed above in Section 4, but, with a two-fold degeneracy. This twofold degeneracy has two important consequences:

First, rather than go from three to six "colors," we may instead maintain the three colors we already have, and label this two-fold degeneracy over $SU(3)_{QCD}$ by two states, say, $\Uparrow, \Downarrow$, and call this new symmetry $SU(2)_W$. That is, SU(6) will break down to $SU(3)_{QCD}$ x $SU(2)_W$ when going from high to low energies, yielding the precise Yang-Mills internal symmetries observed in nature (deferring, until Section 8, discussion of weak parity violation). Now, $SU(2)_W$ has its foundation in exclusion, based on the need to provide exclusion for all six fermions in $P^{\sigma\mu\nu\tau}$. This exclusion, also, originates in the spacetime properties of the baryons and the currents which populate them.

Second, the remnants of this degeneracy will appear in the tendency of the six different fermions states of SU(6), namely, the $(R\Uparrow, G\Uparrow, B\Uparrow, R\Downarrow, G\Downarrow, B\Downarrow)$ combinations, to cluster into two baryons of three fermions each, at low energy. When the clustering separates out into $(R\Uparrow, G\Uparrow, B\Downarrow), (R\Downarrow, G\Downarrow, B\Uparrow)$, one ends up with a deuteron. This too, precisely accords with what is observed in nature, and would explain why non-isotopic nuclei, built up out of deuteron pairs, are very predominant in nature. In this light, the fourth rank Yang-Mills magnetic object $P^{\sigma\mu\nu\tau}$ is best thought of, not as a "string baryon," but rather, as a dibaryon $P^{\sigma\mu\nu\tau}$, of which the deuteron is the most common special case. At low energies, the six exclusive states of $(R\Uparrow, G\Uparrow, B\Uparrow, R\Downarrow, G\Downarrow, B\Downarrow)$, split into various combinations of two three-fermion baryons each. Non-isotopic nuclei are then built up out of a plurality of $P^{\sigma\mu\nu\tau}$, but at low energies, are witnessed predominantly as dibaryons, e.g., deuterons consisting of two baryons $P^{\sigma\mu\nu}$ (see, e.g., [21], [22], [23] which consider six-quark bags and dibaryons.) In this way, the existence of deuterons = pn = 3 quarks x 2 baryons throughout the nuclear world, as well as other dibaryons, is also seen to directly mirror the low energy Yang-Mills phenomenology $SU(3)_{QCD}$ x $SU(2)_W$.

All of this, may provide a unified basis for strong and weak interactions.

If we use $\Rightarrow$ to designate the high-to-low energy transition from $P^{\sigma\mu\nu\tau}$ to $P^{\sigma\mu\nu}$, and $J^{\mu\nu}$ to J^{μ}, together with the associated internal symmetries, then we may summarize the above discussion by writing:

$$SU(6)\left(P^{\sigma\mu\nu\tau}, J^{\mu\nu}\right) \Rightarrow SU(3)_{QCD} \times SU(2)_{W}\left(P^{\sigma\mu\nu}, J^{\mu}\right) = baryon \times 2 = dibaryon, e.g., deuteron\,. \quad (6.2)$$

Now that we have a basis for $SU(2)_W$ based on exclusion, we can introduce the I^i, $i = 1,2,3$ of the weak interaction operating on fermion wavefunctions $\psi^T = (\Uparrow \quad \Downarrow)$, and with this, we have a foundation upon which to discuss specific baryons such as the proton and neutron, specific mesons such as the $\pi^0, \pi^{\pm}$, and even specific dibaryons such as the deuteron.

Having gone from Yang-Mills magnetic charges to strong baryons and mesons and $SU(3)_{QCD}$, and then on to protons and neutrons and deuterons pions and $SU(3)_{QCD}$ x $SU(2)_W$ using the pedagogical bridge of exclusion, let us now ask the next question: what about atoms themselves? Is there some way based on similar "counting" arguments and exclusion principles, to progress forward to represent an entire atom, at least for the simplest atoms? Put in pedagogical context: "What is the exclusion principle, based on the spacetime properties of baryons, which forces atoms into existence?" This involves yet another important step, because in asking about atoms, we need electrons as well as quarks, and so, are now asking about the quark / lepton relationship.

The other question which arises, is whether there is an explanation for fermion generation replication to be obtained from the pedagogical approach of extracting internal symmetries from the spacetime properties of baryons via the bridge of exclusion. That is, if this pedagogy holds, then it should also be possible to extract quantum numbers such as *C* and *S* for the charmed and strange quarks, as an example, from the spacetime properties of "regular," as well as "plus one rank," baryons and currents.

7. Lepto-Quark Unification, Foundations of Atomic Structure, and Superconductivity

In the prior section, we assumed that there is a simultaneous transition from $P^{\sigma\mu\nu\tau}$ to $P^{\sigma\mu\nu}$, and from $J^{\mu\nu}$ to J^{μ}. Let us relax this assumption, and ask what would happen – based again on spacetime index counting alone – if we were to populate a fourth rank string baryon $P^{\sigma\mu\nu\tau}$, with ordinary, first rank currents J^{μ}. Similarly, we ask, what would happen if we were to populate an ordinary third rank $P^{\sigma\mu\nu}$, with a second rank string currents $J^{\mu\nu}$. That is, we examine how many fermions arise from each of the combinations $\left(P^{\sigma\mu\nu}, J^{\mu}\right)$, $\left(P^{\sigma\mu\nu\tau}, J^{\mu\nu}\right)$, $\left(P^{\sigma\mu\nu\tau}, J^{\mu}\right)$, and $\left(P^{\sigma\mu\nu}, J^{\mu\nu}\right)$, where (P, J) generally means "populate baryon *P* with currents *J* and then apply exclusion." The $\left(P^{\sigma\mu\nu}, J^{\mu}\right)$ combination was the focus of the discussion in sections 4 and 5, and led via exclusion to $SU(3)_{QCD}$, confinement, and short-range mesons. The $\left(P^{\sigma\mu\nu\tau}, J^{\mu\nu}\right)$ combination was examined in section 6 as summarized in (6.2), and led via exclusion to SU(6) $\supset$ $SU(3)_{QCD}$ x $SU(2)_W$ and a possible understanding of why deuterons are such a fundamental nuclear building block, effectively mirroring $SU(3)_{QCD}$ x $SU(2)_W$. We shall now show how $\left(P^{\sigma\mu\nu\tau}, J^{\mu}\right)$ leads to leptons and atoms, and $\left(P^{\sigma\mu\nu}, J^{\mu\nu}\right)$ to generation replication.

The $\left(P^{\sigma\mu\nu\tau}, J^{\mu}\right)$ combination, that is, fourth rank $P^{\sigma\mu\nu\tau}$ populated with first rank J^{μ}, would contain 4 = C(4,1) distinct currents based on spacetime indexes, and hence, four fermions $\psi_{(\mu)}$, $\psi_{(\nu)}$, $\psi_{(\sigma)}$, $\psi_{(\tau)}$ in need of exclusion. The Yang-Mills group of choice is now SU(4). Because we want this SU(4) to break down to $SU(3)_{QCD}$ upon the $P^{\sigma\mu\nu\tau}$ to $P^{\sigma\mu\nu}$ transition, we retain the three colors *R*, *G*, *B* already in place for $SU(3)_{QCD}$. That is, we establish SU(4) such that SU(4) ⊃ $SU(3)_{QCD}$. With SU(4), comes an additional diagonal generator $T^{15} = \frac{1}{2}\lambda^{15}$, which, like the other λ^i for $SU(3)_{QCD}$, is to be normalized such that $Tr(\lambda^{15})^2 = 2$. If we then assign $B - L = (\sqrt{6}/3)\lambda^{15}$ where *B* is baryon number and *L* is lepton number, then we may designate this fourth "color" of fermion as *L*, and establish the $SU(4)_{\text{lepto-quark}}$ wavefunction $\psi^T = (L \quad R \quad G \quad B)$ (see, e.g., [24], section 12.2). This fourth rank $P^{\sigma\mu\nu\tau}$, but with first rank currents J^μ, now contains four fermions, three of which are quarks, and one of which is a lepton. When this $P^{\sigma\mu\nu\tau}$ with J^μ breaks down to $P^{\sigma\mu\nu}$ with J^μ, the three quarks will cluster into a single baryon, while the lepton manifest separately. The resulting symmetry is now $SU(3)_{QCD}$ + $U(1)_{\text{lepton}}$. Similarly to the deuteron discussion earlier, we then should also come to expect that we will observe natural systems consisting of one baryon and one lepton. That is, if $SU(6)(P^{\sigma\mu\nu\tau}, J^{\mu\nu})$ is the natural "template" for a deuteron, then $SU(4)(P^{\sigma\mu\nu\tau}, J^\mu)$ is the natural "template" for an atom. To summarize:

$$SU(4)_{lepto-quark}(P^{\sigma\mu\nu\tau}, J^\mu) \Rightarrow SU(3)_{QCD} + U(1)_{lepton}(P^{\sigma\mu\nu}, J^\mu) = baryon + lepton = atom. \qquad (7.2)$$

There is a wealth of evidence that this phenomenology is also fully in accordance with nature. When we observe atoms, we are observing the low-energy manifestation of $SU(4)(P^{\sigma\mu\nu\tau}, J^\mu)$.

Finally, if a fourth rank baryon $P^{\sigma\mu\nu\tau}$ may contain three quarks and a lepton without the lepton being separate from the remaining quarks, one may be able to explain the underlying nuclear mechanism for superconductivity. For a non-superconducting material, one encounters resistance, presumably, because electrons traveling through that material cannot go "through" the nucleons of that material, but must go "around" the nucleons and so encounter "friction," i.e., resistance, by virtue of numerous "collisions" with the nucleons. Now, suppose we "cool" the material below a threshold temperature T_C such that the electrons then travel without resistance, which is, in a word, superconductivity. If this cooling were to restore $SU(4)_{lepto-quark}(P^{\sigma\mu\nu\tau}, J^\mu)$ from $SU(3)_{QCD} + U(1)_{lepton}(P^{\sigma\mu\nu}, J^\mu)$, then the electron, being just another quark in a four-fermion nucleon, would become part of the nucleon, and not be separate therefrom. Differently put, an atom becomes a plurality of four-fermion baryons. Collisions would be eliminated, and the current would then flow without resistance. Thus, if we regard (7.2) as describing a transition which takes place as one goes supplies heat, that is, if we regard $SU(4)_{lepto-quark}$ as a low-temperature symmetry, then (7.2) might possibly be considered as the underlying nuclear mechanism for superconductivity, as well as a possible connection with thermodynamics.

This becomes especially interesting in light of the fact that only mesons – not individual fermions – can travel through a confinement surface, as developed in section 5. Thus, once an electron becomes a fourth color of quark at low temperature, then even if it can coexist with quarks inside of a single four-fermion baryon, the electrons would still have to form leptonic "mesons," e^+e^-, that is, electron / positron pairs, in order to flow through the confinement surface. Thus, one would expect electron "pairing," as well as the presence of positron "holes," to be an essential signature of a superconducting current flow. Intriguingly, Cooper pairing [25], and Dirac holes [26], have long been part of superconducting theory at the electronic level. Finally, for an $SU(4)_{lepto-quark}$ "baryon atom," there is also nothing to prevent current flows of quark / antiquark pairs and even mixed quark / positron and electron / antiquark pairs. Therefore, one would expect to observe fractional current flows with electrical charges of $\pm\frac{1}{3}$ or $\pm\frac{2}{3}$ as well as ± 1. This could underlie the fractional quantum Hall effect. [27]

8. The Generation Mystery, Fermion Phenomenology, and Electric Charge Generator

Now, let's turn to $\left(P^{\sigma\mu\nu}, J^{\mu\nu}\right)$, i.e., a regular third rank baryon $P^{\sigma\mu\nu}$ populated with second rank string currents $J^{\mu\nu}$. Here, we have 3=C(3,2) index combinations, and so three distinct currents and fermions. The fermions are: $\psi_{(\sigma\mu)}$, $\psi_{(\sigma\nu)}$, $\psi_{(\mu\nu)}$ in contrast to the $\psi_{(\mu)}$, $\psi_{(\nu)}$, $\psi_{(\sigma)}$ of section 4 for which 3=C(3,1). We again force exclusion, but not the same R, G, B exclusion of $SU(3)_{QCD}$. We need to find a different "3." The one other piece of particle phenomenology not yet addressed is generation replication. We do observe three generations. And, here, we have an additional SU(3) symmetry in need of a phenomenological association. Let us therefore, enforce exclusion by assigning these $\psi_{(\sigma\mu)}$, $\psi_{(\sigma\nu)}$, $\psi_{(\mu\nu)}$ fermions to the states e,μ,τ, and regard this as an $SU(3)_{GEN}$ symmetry of generation replication. Again, we have applied the pedagogical method of using exclusion to bridge spacetime and internal symmetries.

If we do this, we can now summarize the various ways of "populating" a baryon, and their associated internal symmetries, using exclusion and "counting," as follows:

For the baryons:

$$\left(P^{\sigma\mu\nu}, J^{\mu}\right) \quad \Rightarrow \quad \psi_{(\mu)},\psi_{(\nu)},\psi_{(\sigma)} \quad \overset{exclusion}{\Rightarrow} \quad 3_{QCD} = (R,G,B); \tag{8.1}$$

For the dibaryons:

$$\left(P^{\sigma\mu\nu\tau}, J^{\mu\nu}\right) \Rightarrow \psi_{(\sigma\mu)},\psi_{(\sigma\nu)},\psi_{(\sigma\tau)},\psi_{(\mu\nu)},\psi_{(\mu\tau)},\psi_{(\nu\tau)} \overset{exclusion}{\Rightarrow} 6 = 3\times 2 = (R,G,B)\times(\Uparrow,\Downarrow); \tag{8.2}$$

For the atoms:

$$\left(P^{\sigma\mu\nu\tau}, J^{\mu}\right) \quad \Rightarrow \quad \psi_{(\mu)},\psi_{(\nu)},\psi_{(\sigma)},\psi_{(\tau)} \quad \overset{exclusion}{\Rightarrow} \quad 4 = (L,R,G,B); \tag{8.3}$$

For the generations:

$$\left(P^{\sigma\mu\nu}, J^{\mu\nu}\right) \quad \Rightarrow \quad \psi_{(\sigma\mu)},\psi_{(\sigma\nu)},\psi_{(\mu\nu)} \quad \overset{exclusion}{\Rightarrow} \quad 3_{GEN} = (e,\mu,\tau); \tag{8.4}$$

With this, we can take the "counting" argument even a step further, and arrive at the complete phenomenology of the elementary fermions, in the following way:

In a four dimensional spacetime, 4! = 4x3x2x1 = 24 describes the number of permutations by which the indexes of a fourth rank tensor can be reordered. We have seen that all of these factors, 4, 3, 2, show up by the various combinatorial arguments that led to (8.1) through (8.4). In general, C(M,N)=M!/(M-N)!N!. Let us now associate the pure numbers 4, 3, and 2 from 4! with $4=(L,R,G,B)$, $3=(e,\mu,\tau)$, and $2=(\Uparrow,\Downarrow)$, noting the origin of each of these numbers in (8.1) through (8.4). We then use $24=4\times 3\times 2=(L,R,G,B)\times(e,\mu,\tau)\times(\Uparrow,\Downarrow)$ to form exactly 24 particles. These are summarized in Figure 2 below:

$$
\begin{array}{c|ccc|ccc}
 & \multicolumn{3}{c|}{\leftarrow 3 = e\mu\tau \rightarrow} & \multicolumn{3}{c}{\leftarrow 3 = e\mu\tau \rightarrow} \\
\uparrow & \nu_e & \nu_\mu & \nu_\tau & e & \mu & \tau \\
4 = & u_R & c_R & t_R & d_R & s_R & b_R \\
LRGB & u_G & c_G & t_G & d_G & s_G & b_G \\
\downarrow & u_B & c_B & t_B & d_B & s_B & b_B \\
 & \multicolumn{6}{c}{\leftarrow 2 = \Uparrow\Downarrow \rightarrow}
\end{array}
$$

Figure 3

We do, in fact, find exactly 24 distinct elementary fermion in nature. It is now seen as more than just happenstance, that 24 = 4! = 4x3x2x1 is so closely associated with the properties of antisymmetric tensors in 4-dimensional spacetime and that there are 24 known fermions which in fact group themselves into the 4x3x2 configuration of Figure 3. Rather, the existence of exactly 24 elementary fermions It is motivated by the fundamental numeric fact that the number 24 = 4! naturally emerges from antisymmetric field theory in a four-dimensional spacetime, and the various ways in which baryons can be populated with lower-rank currents. The internal symmetries of the elementary fermions, thus have a direct and fundamental connection to the symmetry of spacetime in antisymmetric field theory, via the pedagogical use of Pauli exclusion as a bridge between spacetime and internal symmetries.

At this point, the only internal symmetry left unexplained is the electric charge $Q=Y+I^3{}_L$. Volovok points out in [24] Section 12.2.2, that once $B-L=(\sqrt{6}/3)\lambda^{15}$ is established, see the previous discussion preceding (7.2), that the weak hypercharge generator *Y*, can be established with a weak right-handed $SU(2)_R$ group $Y=\frac{1}{2}(B-L)+I^3{}_R$, and therefore, the electric charge generator is $Q=\frac{1}{2}(B-L)+I^3{}_R+I^3{}_L$. In this circumstance, to get to *Q*, and hence to the phenomenological $SU(3)_{QCD}$ x $SU(2)_L$ x $U(1)_Y$, one needs to introduce the $SU(2)_W$ degeneracy discussed in Section 6 as a left-right symmetric internal symmetry, and then uncover how to break this chiral symmetry at low energies so that only $SU(2)_L$ is observed. We leave the question of chiral symmetry breaking for a separate undertaking.

9. Baryon Wavefunctions

Finally, returning to the regular baryon $P^{\mu\nu\sigma}$, one may ask, "how might the wavefunction for a baryon be related to the baryon source density $P^{\mu\nu\sigma}$?" Given that $J^\mu=T^i(\overline{\psi}T_i\gamma^\mu\psi)$, see (3.5), relates the Dirac wavefunction ψ for a fermion to the first-rank current source density J^μ, one may try to construct a similar expression relating a baryon wavefunction Ψ to the third-rank baryon source density $P^{\mu\nu\sigma}$. Because $P^{\mu\nu\sigma}$ is fully antisymmetric, we will need, however, rather than the Dirac γ^μ, to employ totally-antisymmetric, third rank combinations of the Dirac γ^μ. First, recognizing that $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and so using a *fifth*-rank, totally antisymmetric Levi-Civita tensor $\varepsilon^{\mu\nu\sigma\tau\lambda}$ with $\varepsilon^{01235} = 1$, we define the "dual" $*\sigma^{\mu\nu\sigma}$ of the usual antisymmetric object $\sigma_{\tau\lambda} = \frac{i}{2}[\gamma_\tau\gamma_\lambda - \gamma_\lambda\gamma_\tau]$ according to:

$$*\sigma^{\mu\nu\sigma} \equiv \frac{1}{2!}\varepsilon^{\mu\nu\sigma\tau\lambda}\sigma_{\tau\lambda} = \frac{i}{4}\varepsilon^{\mu\nu\sigma\tau\lambda}[\gamma_\tau\gamma_\lambda - \gamma_\lambda\gamma_\tau]. \quad (9.1)$$

The $*\sigma^{\mu\nu\sigma}$, so-defined, are totally antisymmetric in all three spacetime indexes. Then, we may *define* Ψ from $P^{\mu\nu\sigma}$, analogously to $J^\mu = T^i\left(\overline{\psi}T_i\gamma^\mu\psi\right)$, according to:

$$T^i\left(\overline{\Psi}T_i * \sigma^{\mu\nu\sigma}\Psi\right) \equiv P^{\mu\nu\sigma}. \quad (9.2)$$

For μ,ν,σ ranging only over the spacetime indexes 0,1,2,3, it is clear from (9.1) and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ that $*\sigma^{\mu\nu\sigma}$ will always lead to an axial vector, containing the commutator of a Dirac matrix γ^5 with a γ^μ such that $\mu \neq 5$. For example, $*\sigma^{012} = \frac{i}{2}[\gamma_3\gamma_5 - \gamma_5\gamma_3] = i\sigma_{35}$. The above discussion in this paragraph illustrates a possible approach for development of baryon wavefunctions. Definitive calculation, beyond the scope of this paper, will be needed to confirm whether these constructions are physically correct, and in accordance with known understandings of baryon wavefunctions.

10. Conclusion

As a result of the foregoing, we conclude that the baryons may well be Yang-Mills magnetic sources, described most simply and compactly in equation (4.2), and represented in the Feynman diagram of Figure 1. The color group $SU(3)_{QCD}$ emerges naturally from the spacetime symmetries of the baryon, by demanding quantum exclusion for the three fermions within a baryon. Quark and gluon confinement, and the existence of short-range mesons mediating strong nuclear interactions, naturally emerge from this analysis. When we consider the possibility of strings, quantum exclusion appears to lead also to the $SU(2)_W$ symmetry of weak interactions, and to an understanding of why baryons may cluster into deuteron pairs. Further consideration leads as well to a possible understanding of the relation between quarks and leptons, and the origins of atoms, as well as the fundamental phenomenology of 24 elementary fermion flavors, in a 24 = 4! = 4x3x2 = D! grouping, in a D=4 dimensional spacetime. An irony is that string theory, often criticized for being unable to predict any experimental results, appears here, together with a careful analysis of magnetic sources in Yang-Mills field theory and the pedagogical application of exclusion, to lay the foundation for explaining a wide range of phenomenology, including the observed strong / weak /electromagnetic $SU(3)_{QCD}$ x $SU(2)_W$ x $U(1)_Y$ interaction phenomenology, nuclear and atomic structure, and the observed pattern of elementary fermions including generation replication.

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[26] A good summary of recent hole superconductor literature can be found at http://www-physics.ucsd.edu/~jorge/hole.html.

[27] See, e.g., http://www.warwick.ac.uk/~phsbm/fqhe.htm: "In high mobility semiconductor heterojunctions the integer quantum Hall effect (IQHE) plateaux are much narrower than for lower mobility samples. Between these narrow IQHE more plateaux are seen at fractional filling factors, especially 1/3 and 2/3. This is the fractional quantum Hall effect (FQHE) whose discovery in 1982 was completely unexpected."